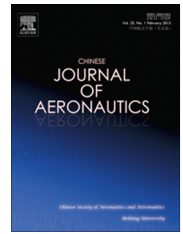




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Adaptive control for attitude coordination of leader-following rigid spacecraft systems with inertia parameter uncertainties

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Abstract This paper studies the leader-following attitude coordination problems of multiple spacecraft in the presence of inertia parameter uncertainties. To achieve attitude coordination in the situation that even the leader's attitude is only applicable to a part of the following spacecraft, a non-linear attitude observer is proposed to obtain an accurate estimation of the leader's attitude and angular velocity for all the followers. In addition, a distributed control scheme based on non-certainty equivalence principle is presented for multiple spacecraft attitude synchronization. With a dynamic scaling, attitude consensus can be achieved asymptotically without any information of the bounds of the uncertain inertia parameters. Furthermore, once the estimations of inertia parameters reach their ideal values, the estimation process will stop and the ideal value of inertia parameter will be held. This is a special advantage of parameter estimation method based on non-certainty equivalence. Numerical simulations are presented to demonstrate that the proposed non-certainty equivalence-based method requires smaller control torque and converges faster compared with the certainty equivalence-based method.

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1. Introduction

Spacecraft formation flying has drawn extensive attention in the last decade due to its higher flexibility and robustness,

greater efficiency and lower fuel consumption.^{1–4} Attitude coordination has been expected to be a necessary and important technology for many space missions, such as the CanX-4&5 primarily supported by Canadian Space and the SULFRO mission concept led by Chinese Academy of Sciences.⁵ Compared with the leaderless case studied in Ref.⁶, the leader-following attitude coordination control is more challengeable since it not only ensures the consensus of the followers' attitude, but also enables the followers' attitude and angular velocity to be in line with those of the leader.^{7,8} If there is only one follower, the leader-following problem degenerates into the attitude tracking problem of spacecraft.^{9,10}

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According to how the followers access the leader's states, the methods for tackling leader-following problem can be classified into decentralized and distributed methods. The decentralized control algorithms assume that the leader's state is accessible to every follower in the group.^{11–16} A passivity-based controller for a group of spacecraft without inertial frame information was developed in Ref.¹³. Meng et al. designed two distributed containment control laws for attitude consensus of spacecraft, including both static and dynamic leaders.¹⁴ The result indicates that the attitude of the followers will converge to the convex hull formed by all leaders. A non-singular fast terminal sliding mode controller was proposed to counteract the impact of external disturbances and actuator failures in Ref.¹⁵ Attitude coordination with communication delays, caused by the distance between satellites, was taken into consideration in Ref.^{16,17}

In practice, it is more realistic that only a part of the followers can access the leader's attitude. Several distributed control laws have been presented to solve this problem in Ref.^{18–22} Ren firstly studied distributed attitude synchronization of the leader-following problem with attitudes represented by modified Rodriguez parameters.¹⁸ Cai and Huang proposed a distributed nonlinear estimation method of leader's state for followers under the assumption that the communication network of the spacecraft systems is undirected and connected.¹⁹ This work was extended to the case without the measurements of angular velocity by employing an angular velocity auxiliary system in Ref.²⁰ Zou et al. also studied the attitude coordination of the leader-following spacecraft systems without angular velocity measurement under undirected communication graph and proposed a finite-time observer to obtain the unmeasurable angular velocity of the leader.²¹ Du extended attitude consensus of leader-following spacecraft systems to the situation with both rigid and flexible spacecraft.²² However, all the above literatures did not consider attitude coordination with uncertain inertia parameters.

Uncertainties of the spacecraft, caused by fuel consumption, appendage deployment and onboard payload transformation, are challenging to spacecraft attitude synchronization. Attitude tracking for single spacecraft with unknown inertia parameter has been investigated in Ref.^{9,10,23–25} However, the development from attitude tracking problem to leader-following problem is nontrivial, especially when the leader's information cannot be directly accessed by all the followers. Several authors have studied the leader-following attitude coordination problem associated with parameter uncertainties.^{26–30} Wu et al. studied the attitude synchronization and tracking problem of spacecraft with parameter uncertainties.^{26,27} In these papers, the uncertainties were assumed to be a known nominal part and an uncertain part. A robust nonlinear controller for both relative position tracking and attitude synchronization with external disturbance and model uncertainty was presented in Ref.²⁸ However, initial values of the inertia parameters were regarded as known constants. Recently, Cai and Huang proposed a distributed control algorithm based on Certainty-Equivalence (CE) for multiple spacecraft systems in the presence of inertia parameter uncertainties.³¹ The authors later extended their research to the condition with both uncertainties and external disturbances.³² Although the asymptotic convergence of the tracking

errors is easy to achieve in the existing literatures, the parameter estimation process is directly affected by tracking error dynamics, which could result in bad transient performance.

The main aim of this paper is to provide a high-performance solution for the problem of attitude coordination for leader-following spacecraft in the presence of inertia parameter uncertainties. Unlike the CE-based approaches in attitude consensus with uncertainties, a distributed control law based on dynamic-scaling-based non-certainty equivalence is proposed. By setting an upper bound for dynamic scaling factor, the proposed method can not only make the convergence rate of attitude coordination faster, but also avoid the bad transient performance compared with CE-based method. In addition, the asymptotic convergence of tracking error under all the possible initial conditions is guaranteed without considering the boundary information of system uncertainties. Moreover, the distributed observer proposed in Ref.²⁰ is modified and applied to overcome the difficulty that the leader's attitude and angular velocity are only accessible to a part of the followers. This distributed observer guarantees that the attitude estimation made by the followers will converge to the leader's attitude, and then all the followers can track the estimation to achieve attitude consensus with the leader.

The following of this paper is organized as follows. The attitude dynamics of spacecraft and problem formulation are briefly described in Section 2. Section 3 introduces a nonlinear distributed observer. The controller is given and its stability analysis is made in Section 4. Simulations are presented in Section 5 to illustrate our results. Conclusions are given in Section 6.

2. Preliminaries and problem formulation

2.1. Algebraic graph theory

In this subsection, algebraic graph theory is briefly introduced to describe the communication networks among the leader-following spacecraft system. A directed graph (or digraph) is represented as $\mathcal{G}(V(\mathcal{G}), E(\mathcal{G}))$, in which $V(\mathcal{G}) = \{v_1, v_2, \dots, v_N\}$ represents a vertex set, and $E(\mathcal{G}) \subset V(\mathcal{G}) \times V(\mathcal{G})$ represents an edge set of ordered pair of vertices. The edge set consists of elements of the form (v_i, v_j) if vertex v_j can get some information from vertex v_i . An adjacency matrix $A(\mathcal{G}) = [a_{ij}] \in \mathbf{R}^{N \times N}$ of digraph $\mathcal{G}$ is defined as: if $(v_i, v_j) \in E(\mathcal{G})$, $a_{ij} > 0$, otherwise $a_{ij} = 0$. With the adjacency matrix $A(\mathcal{G})$, the Laplacian matrix $L(\mathcal{G}) = [l_{ij}] \in \mathbf{R}^{N \times N}$ of the digraph $\mathcal{G}$ is given as: if $i = j$, $l_{ij} = \sum_{j=1}^N a_{ij}$, otherwise $l_{ij} = -a_{ij}$.

In the leader-following spacecraft system, $v_1, v_2, \dots, v_N$ represent the N followers and $\mathcal{G}$ represents the communicating relations between all following spacecraft. The leader is denoted as v_0 and the edges between the leader and all followers are denoted as E_{lf} . Then, the corresponding communication networks of leader-following spacecraft system can be represented as $\bar{\mathcal{G}}(V(\bar{\mathcal{G}}), E(\bar{\mathcal{G}}), A(\bar{\mathcal{G}}))$ in which $V(\bar{\mathcal{G}}) = V(\mathcal{G}) \cup v_0$ is the vertex set, $E(\bar{\mathcal{G}}) = E(\mathcal{G}) \cup E_{lf}$ is the edge set and $A(\bar{\mathcal{G}}) = [a_{ij}] \in \mathbf{R}^{(N+1) \times (N+1)}$, $i, j \in \{0, 1, \dots, N\}$ is the adjacency matrix of $\bar{\mathcal{G}}$. The adjacency matrix between the leader and all

following spacecraft is denoted by $\mathcal{B}(\bar{\mathcal{G}}) = \text{diag}\{a_{10}, \dots, a_{10}\} \in \mathbf{R}^{N \times N}$, where $a_{i0} > 0$ if the i -th follower can access the state of the leader, otherwise $a_{i0} = 0$. Since there is no edge from the follower to the leader, $a_{0i} = 0$ for all $i = 1, 2, \dots, N$. If not otherwise specified, matrix $\mathbf{L}(\bar{\mathcal{G}})$ and $\mathcal{B}(\bar{\mathcal{G}})$ are abbreviated as $\mathbf{L}$ and $\mathbf{B}$ in the following sections respectively.

Assumption 1. The communication digraph $\bar{\mathcal{G}}$ of leader-following spacecraft system contains a spanning tree, in which the leader spacecraft (v_0 in $\bar{\mathcal{G}}$) is the root.

Remark 1. Assumption 1 is the weakest condition to guarantee the accessibility of the leader's attitude information by all the followers.

2.2. Problem formulation of attitude synchronization

Consider a system containing N rigid spacecraft with the following attitude dynamic equations:

$$\begin{cases} \dot{\mathbf{q}}_i = \frac{1}{2} \mathbf{q}_i \odot \mathbf{q}(\omega_i) \\ \mathbf{J}_i \dot{\omega}_i = -\omega_i^\times \mathbf{J}_i \omega_i + \mathbf{u}_i \end{cases} \quad i = 1, 2, \dots, N \quad (1)$$

where $\mathbf{J}_i \in \mathbf{R}^{3 \times 3}$ is the symmetric and positive definite inertia matrix of the i -th spacecraft. A quaternion $\mathbf{q} = [\bar{\mathbf{q}}, \hat{\mathbf{q}}]^\top \in \mathbf{R} \times \mathbf{R}^3$ is denoted as $\mathbf{q} \in \mathbf{Q}$, and a unit quaternion is denoted as $\mathbf{q} \in \mathbf{Q}_u$ if it satisfies the constraint $\bar{\mathbf{q}}^2 + \hat{\mathbf{q}}^\top \hat{\mathbf{q}} = 1$. $\mathbf{q}_i \in \mathbf{Q}_u$ is a unit quaternion describing the orientation of the body frame $\mathcal{B}_i$ of the i -th spacecraft with respect to the inertial frame; $\omega_i = [\omega_{ix}, \omega_{iy}, \omega_{iz}]^\top \in \mathbf{R}^3$ denotes the attitude velocity of the body frame $\mathcal{B}_i$ of the i -th spacecraft relative to the inertial frame $\mathcal{I}$; $\mathbf{u}_i \in \mathbf{R}^3$ denotes the control torque of the i -th spacecraft. The variable quantities ω_i , $\mathbf{J}_i$, $\mathbf{u}_i$ are all expressed in $\mathcal{B}_i$. The operator $q(\cdot)$ converts a vector $\mathbf{x} \in \mathbf{R}^3$ to a quaternion $q(\mathbf{x}) \equiv [0, \mathbf{x}^\top]^\top \in \mathbf{R} \times \mathbf{R}^3$. The notation $\omega^\times$ for $\forall \omega = [\omega_1, \omega_2, \omega_3] \in \mathbf{R}^3$ represents the following skew symmetric matrix $\omega^\times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$. The operator $\odot$ represents the quaternion product: for $\mathbf{q}_i, \mathbf{q}_j \in \mathbf{Q}$,

$$\mathbf{q}_i \odot \mathbf{q}_j = \begin{bmatrix} \bar{\mathbf{q}}_i \bar{\mathbf{q}}_j - \hat{\mathbf{q}}_i^\top \hat{\mathbf{q}}_j \\ \bar{\mathbf{q}}_i \hat{\mathbf{q}}_j + \bar{\mathbf{q}}_j \hat{\mathbf{q}}_i + \hat{\mathbf{q}}_i^\times \hat{\mathbf{q}}_j \end{bmatrix}.$$

The reference attitude $\mathbf{q}_0$ of the leader is generated according to the following system:

$$\dot{\mathbf{q}}_0 = \frac{1}{2} \mathbf{q}_0 \odot \mathbf{q}(\omega_0) \quad (2)$$

where $\mathbf{q}_0 = [\bar{\mathbf{q}}_0, \hat{\mathbf{q}}_0]^\top \in \mathbf{Q}_u$ is an unit quaternion describing the orientation of the body frame $\mathcal{B}_0$ of the leader spacecraft relative to inertial frame $\mathcal{I}$, and $\omega_0 \in \mathbf{R}^3$ represents attitude velocity of the body frame $\mathcal{B}_0$ of leader spacecraft with respect to the inertial frame $\mathcal{I}$. In particular, ω_0 is assumed as a bounded function with bounded $\dot{\omega}_0$ and $\ddot{\omega}_0$.

In view of Eqs. (1) and (2), the attitude tracking errors between the i th spacecraft and the leader are given as

$$\begin{cases} \epsilon_i = \mathbf{q}_0^{-1} \odot \mathbf{q}_i \\ \tilde{\omega}_i = \omega_i - C(\epsilon_i) \omega_0 \end{cases} \quad (3)$$

where $\epsilon_i \in \mathbf{Q}_u$ and $C(\epsilon_i) = (\bar{\epsilon}_i^2 - \hat{\epsilon}_i^\top \hat{\epsilon}_i) \mathbf{I}_3 + 2\bar{\epsilon}_i \hat{\epsilon}_i^\top - 2\hat{\epsilon}_i \hat{\epsilon}_i^\times$ represent the unit quaternion attitude tracking error and direction cosine matrix between frame $\mathcal{B}_i$ and $\mathcal{B}_0$, respectively; $\mathbf{I}_3 \in \mathbf{R}^{3 \times 3}$ denotes the identity matrix; $\tilde{\omega}_i$ represents the angular velocity tracking error between ω_i and ω_0 . From the definition above, the attitude tracking error dynamics are obtained as follows:

$$\begin{aligned} \dot{\epsilon}_i &= \frac{1}{2} \epsilon_i \odot \mathbf{q}(\tilde{\omega}_i) \\ \mathbf{J}_i \dot{\tilde{\omega}}_i &= -\omega_i^\times \mathbf{J}_i \omega_i + \mathbf{J}_i (\tilde{\omega}_i^\times C(\epsilon_i) \omega_0) - C(\epsilon_i) \dot{\omega}_0 + \mathbf{u}_i. \end{aligned} \quad (4)$$

The purpose of this paper is to solve the following problem associated with parameter uncertainties.

Problem 1. Given systems (1) and (2), under Assumption 1, design a distributed control law such that, for all $\omega_i(0) \in \mathbf{R}^3$ and all $\mathbf{q}_i(0) \in \mathbf{Q}_u$,

$$\lim_{t \rightarrow \infty} \epsilon_i(t) = \mathbf{0} \quad \lim_{t \rightarrow \infty} \tilde{\omega}_i(t) = \mathbf{0}, \quad i = 1, 2, \dots, N.$$

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Assumption 2. The leader's angular velocity $\omega_0(t)$ is generated the same as¹⁹

$$\dot{\omega}_0 = \mathbf{S} \omega_0 \quad (5)$$

where $\omega_0 \in \mathbf{R}^3$, $\mathbf{S} \in \mathbf{R}^{3 \times 3}$, and system (5) is assumed to be marginally stable.

Assumption 3. The leader's angular velocity $\omega_0(t)$ satisfies the persistent excitation inequality²⁹

$$\int_t^{t+T_0} \omega_0(\tau) \omega_0(\tau)^\top d\tau \geq \ell \mathbf{I}_n \quad \forall t \geq t_0 \quad (6)$$

for positive constants t_0 , T_0 and ℓ .

Remark 2. Assumption 2 implies that the leader's angular velocity ω_0 is bounded. Additionally, according to system (2), $\mathbf{q}_0$ is bounded too. The model of similar form has been widely used in Ref.^{19,20,30–32} However, these literatures all assume that the matrix $\mathbf{S}$ is known to all followers to make the follower be able to generate the same angular acceleration as the leader. To remedy the unknown of matrix $\mathbf{S}$, an estimation of $\mathbf{S}$ is proposed based on Immersion & Invariance (I&I) theory with Assumption 3. It is noted that persistent excitation is widely used to ensure the convergence of parameter.

2.3. Lemmas

To prove the main result in this paper, we need the following lemmas:

Lemma 1. The matrix $\mathbf{H} = \mathbf{L} + \mathbf{B}$ is positive stable if and only if digraph $\bar{\mathcal{G}}$ contains a spanning tree with the leader spacecraft as the root.³³

Lemma 2. Let $\mathbf{x}_1 \in \mathbf{Q}_u$, $\mathbf{x}_2 \in \mathbf{Q}$, $\mathbf{x}_3 = \mathbf{x}_1 \odot \mathbf{x}_2$ and $\mathbf{x}_4 = \mathbf{x}_2 \odot \mathbf{x}_1$. Then $\mathbf{x}_2^\top \mathbf{x}_2 = \mathbf{x}_3^\top \mathbf{x}_3 = \mathbf{x}_4^\top \mathbf{x}_4$.²⁰

3. Distributed observer

Since the attitude of the leader is not available to all the following spacecraft, a distributed observer is proposed to estimate the leader's attitude for all other spacecraft. To estimate the angular velocity and angular acceleration of the leader, we define a dynamic compensator similar as¹⁹

$$\begin{cases} \dot{\eta}_i = \frac{1}{2}\eta_i \odot q(\xi_i) + \eta_{di} \\ \dot{\xi}_i = \hat{S}_i \xi_i + \xi_{di} \end{cases} \quad (7)$$

where

$$\begin{cases} \eta_{di} = v_1 \sum_{j=0}^N a_{ij}(\eta_j - \eta_i) \\ \xi_{di} = v_2 \sum_{j=0}^N a_{ij}(\xi_j - \xi_i) \end{cases} \quad (8)$$

and $\eta_i \in \mathbf{Q}$, $\xi_i \in \mathbf{R}^3$, $i = 1, 2, \dots, N$ represent the leader's attitude and angular velocity estimated by the i -th follower, respectively; $\eta_0 = \mathbf{q}_0$, $\xi_0 = \omega_0$, v_1, v_2 are positive real numbers, and $\hat{S}_i$ denotes the estimation of $\mathbf{S}$ by the i -th spacecraft. Let $\eta = [\eta_1^T, \eta_2^T, \dots, \eta_N^T]^T$, $\xi = [\xi_1^T, \xi_2^T, \dots, \xi_N^T]^T$, $\omega_S = \mathbf{1}_N \otimes \omega_0$, $\hat{\mathbf{S}} = \text{diag}\{\hat{S}_1, \hat{S}_2, \dots, \hat{S}_N\}$ and $\bar{\mathbf{S}} = \mathbf{I}_N \otimes \mathbf{S}$, where $\mathbf{1}_N \in \mathbf{R}^N$ denotes the vector of all ones, and $\mathbf{I}_N \in \mathbf{R}^{N \times N}$ denotes the identity matrix. In view of these definitions, Eqs. (5) and (7) can be rewritten in a compact form as

$$\begin{cases} \dot{\omega}_S = \bar{\mathbf{S}}\omega_S \\ \dot{\xi} = \hat{\mathbf{S}}\xi - (v_2 \mathbf{L} \otimes \mathbf{I}_3)\xi - (v_2 \mathbf{B} \otimes \mathbf{I}_3)(\xi - \omega_S) \end{cases} \quad (9)$$

For $i = 1, 2, \dots, N$, let $\zeta_i = \eta_i - \mathbf{q}_0$, $\varsigma_i = \xi_i - \omega_0$ denote the attitude and angular velocity error between the leader and the followers' estimation, $\zeta = [\zeta_1^T, \zeta_2^T, \dots, \zeta_N^T]^T$ and $\varsigma = [\varsigma_1^T, \varsigma_2^T, \dots, \varsigma_N^T]^T$, and then, the time derivative of ς is given as

$$\begin{aligned} \dot{\varsigma} &= -(v_2 \mathbf{H} \otimes \mathbf{I}_3)\varsigma + \hat{\mathbf{S}}\varsigma - \bar{\mathbf{S}}\omega_S \\ &= -(v_2 \mathbf{H} \otimes \mathbf{I}_3)\varsigma + \hat{\mathbf{S}}\varsigma - \hat{\mathbf{S}}\omega_S + \hat{\mathbf{S}}\omega_S - \bar{\mathbf{S}}\omega_S \\ &= -(v_2 \mathbf{H} \otimes \mathbf{I}_3)\varsigma + \hat{\mathbf{S}}\varsigma + \tilde{\mathbf{S}}\omega_S \\ &= \mathbf{E}\varsigma + \tilde{\mathbf{S}}\omega_0 \end{aligned} \quad (10)$$

where $\mathbf{E} = -(v_2 \mathbf{H} \otimes \mathbf{I}_3) + \hat{\mathbf{S}}$, $\tilde{\mathbf{S}} = \hat{\mathbf{S}} - \bar{\mathbf{S}}$. By Lemma 1, under Assumptions 1 and 2, $\mathbf{H}$ is positive definite and $\mathbf{E}$ is Hurwitz.

Since $\mathbf{S}$ is marginally stable, let $\theta_S = [-S_{11}, -S_{22}, -S_{33}, -S_{32}, -S_{13}, -S_{21}]^T$ denotes the elements of matrix $\mathbf{S}$, and then it can be easily verified that $\mathbf{S}\mathbf{x} = \mathbf{Q}(\mathbf{x})\theta_S$, $\mathbf{x} = [x_1, x_2, x_3]^T \in \mathbf{R}^3$, where

$$\mathbf{Q}(\mathbf{x}) = \begin{bmatrix} -x_1 & 0 & 0 & 0 & -x_3 & x_2 \\ 0 & -x_2 & 0 & x_3 & 0 & -x_1 \\ 0 & 0 & -x_3 & -x_2 & x_1 & 0 \end{bmatrix}$$

We denote $\bar{\theta}_S = \mathbf{1}_N \otimes \theta_S$ and $\hat{\theta}_S = [\hat{\theta}_{S_1}^T, \hat{\theta}_{S_2}^T, \dots, \hat{\theta}_{S_N}^T]^T$. By defining $\hat{\theta}_S = \gamma_S + \beta_S(\xi, \omega_0)$ as the estimation of $\bar{\theta}_S$, the error dynamics of $\hat{\theta}_S = \theta_S - \hat{\theta}_S$ can be written as

$$\begin{aligned} \dot{\hat{\theta}}_S &= \dot{\theta}_S - \dot{\hat{\theta}}_S \\ &= \dot{\gamma}_S + \dot{\beta}_S \\ &= \dot{\gamma}_S + \frac{\partial \beta_S}{\partial \xi} \dot{\xi} + \frac{\partial \beta_S}{\partial \omega_S} \dot{\omega}_S \\ &= \dot{\gamma}_S + \frac{\partial \beta_S}{\partial \xi} \dot{\xi} + \frac{\partial \beta_S}{\partial \omega_S} \mathbf{S}\omega_S \end{aligned} \quad (11)$$

Let γ_S be assigned as

$$\dot{\gamma}_S = -\frac{\partial \beta_S}{\partial \xi} \dot{\xi} + \frac{\partial \beta_S}{\partial \omega_S} \hat{\mathbf{S}}\omega_S \quad (12)$$

and thus, the dynamics of the estimation error $\tilde{\theta}_S$ is further reduced to

$$\dot{\tilde{\theta}}_S = -\frac{\partial \beta_S}{\partial \omega_S} (\hat{\mathbf{S}} - \bar{\mathbf{S}})\omega_S = -\frac{\partial \beta_S}{\partial \omega_S} \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S \quad (13)$$

where $\bar{\mathbf{Q}}(\omega_0) = [\mathbf{I}_N \otimes \mathbf{Q}(\omega_0)]$. However, $\bar{\mathbf{Q}}(\omega_0)$ in Eq. (12) may not be a Jacobian matrix, and the analytical solution of $\partial \beta_S / \partial \omega_S = \rho_S \bar{\mathbf{Q}}(\omega_S)$ cannot be obtained directly.

Next, we define the estimation β_S as

$$\beta_S = \rho_S [\hat{q}_1^T, \hat{q}_2^T, \dots, \hat{q}_N^T]^T \quad (14)$$

where

$$\hat{q}_i = \mathbf{Q}_1(\xi_i)\omega_S(3i-2) + \mathbf{Q}_2(\xi_i)\omega_S(3i-1) + \mathbf{Q}_3(\xi_i)\omega_S(3i) \in \mathbf{R}^6, \quad i = 1, \dots, N$$

and $\mathbf{Q}_1(\xi_i), \mathbf{Q}_2(\xi_i), \mathbf{Q}_3(\xi_i) \in \mathbf{R}^6$ are the column vectors of $\mathbf{Q}^T(\xi_i)$.

Lemma 3. Considering systems (2) and (5), and the estimations given in Eqs. (7), (11), (12) and (14) with Assumptions 1–3, if $v_1, v_2 > 0$, $\mathbf{q}_0(0) \in \mathbf{Q}_u$, $\omega_0(0) \in \mathbf{R}^3$, $\eta_i(0) \in \mathbf{Q}$, $\xi_i(0) \in \mathbf{R}^3$, then $\eta_i(t)$ and $\xi_i(t)$ exist, are bounded for all $t > 0$, and satisfy $\lim_{t \rightarrow \infty} (\hat{S}_i \xi_i(t) - \mathbf{S} \xi_i(t)) = \mathbf{0}$, $\lim_{t \rightarrow \infty} (\eta_i(t) - \mathbf{q}_0(t)) = \mathbf{0}$ and $\lim_{t \rightarrow \infty} (\xi_i(t) - \omega_0(t)) = \mathbf{0}$ exponentially for all $i = 1, 2, \dots, N$.

Remark 3. Assumption 1 is the premise of the establishment of Lemma 3 and it ensures that all the followers can access the leader's state. Then, the followers can utilize the distributed observer given in Eq. (7) to estimate the leader's attitude, angular velocity and angular acceleration. Lemma 3 indicates that the estimation η_i and ξ_i will converge to the leader's attitude $\mathbf{q}_0$ and angular velocity ω_0 , respectively. In addition, Remark 1 shows that $\mathbf{q}_0$ and ω_0 are bounded and then η_i and ξ_i are also bounded.

Since the leader's state might not be accessible to all the followers, the error Eqs. (3) and (4) cannot be employed directly. Instead, for each follower, the error signal is defined as^{20,31}

$$\begin{cases} e_i = \eta_i^* \odot \mathbf{q}_i \\ \bar{\omega}_i = \omega_i - \mathbf{C}(e_i)\xi_i + k_{l_i}\hat{e}_i \end{cases} \quad (15)$$

where $k_{l_i} > 0$ denotes a control gain, $e_i \in \mathbf{Q}$ is the attitude error, $\bar{\omega}_i \in \mathbf{R}^3$ is the angular velocity error, and $\mathbf{C}(e_i) = (\hat{e}_i^2 - \hat{e}_i^T \hat{e}_i)\mathbf{I} + 2\hat{e}_i \hat{e}_i^T - 2\hat{e}_i \hat{e}_i^*$ denotes the corresponding rotation matrix. Differentiating Eq. (15) leads to the following error system:

$$\begin{aligned}
\dot{\mathbf{e}}_i &= \frac{1}{2} \mathbf{e}_i \odot q(\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i) + \mathbf{e}_{1i} \\
\mathbf{J}_i \dot{\hat{\omega}}_i &= -\omega_i^\times \mathbf{J}_i \omega_i \\
&+ \mathbf{J}_i [(\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i)^\times \mathbf{p}_i - \mathbf{p} \mathbf{d}_i + \frac{1}{2} k_{1i} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) (\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i)] + \mathbf{e}_{2i} + \mathbf{u}_i
\end{aligned} \quad (16)$$

where $\mathbf{p}_i = \mathbf{C}(\mathbf{e}_i) \xi_i$, $\mathbf{p} \mathbf{d}_i = \mathbf{C}(\mathbf{e}_i) \hat{\mathbf{S}}_i \xi_i$ and

$$\begin{aligned}
\mathbf{e}_{1i} &= \frac{1}{2} (\mathbf{e}_i^\top \mathbf{e}_i - 1) q(\xi_i) \odot \mathbf{e}_i + \eta_{di}^* \odot \mathbf{q}_i \\
\mathbf{e}_{2i} &= -\mathbf{J}_i (\mathbf{e}_{3i} \xi_i + \mathbf{C}(\mathbf{e}_i) \xi_{di} - k_{1i} \hat{\mathbf{e}}_{1i} + \mathbf{C}(\mathbf{e}_i) \tilde{\mathbf{S}}_i \xi_i) \\
\mathbf{e}_{3i} &= 2\bar{\mathbf{e}}_i \bar{\mathbf{e}}_{1i} \mathbf{I}_3 - 2\hat{\mathbf{e}}_i^\top \bar{\mathbf{e}}_{1i} \mathbf{I}_3 + 2\bar{\mathbf{e}}_{1i} \hat{\mathbf{e}}_i^\top + 2\hat{\mathbf{e}}_i \hat{\mathbf{e}}_i^\top - 2\bar{\mathbf{e}}_{di} \hat{\mathbf{e}}_i^\times - 2\bar{\mathbf{e}}_i \hat{\mathbf{e}}_{1i}^\times
\end{aligned} \quad (17)$$

Remark 4. Eq. (4) represents the real errors between the followers' attitude and that of the leader; however, Eq. (16) represents the errors between the followers' attitude and the estimated attitude of the leader by the i -th follower. Lemma 2 ensures that the estimated attitude of the leader by the followers is convergent to the leader's attitude. Therefore, the convergence of system (16) guarantees the convergence of system (4). In addition, Lemma 3 indicates $\tilde{\mathbf{S}}_i \xi_i \rightarrow \mathbf{0}$ exponentially as $t \rightarrow \infty$, and then by Remark 3.2 in Ref.²⁰, $\mathbf{e}_{1i}$, $\mathbf{e}_{2i}$, $\mathbf{e}_{3i} \rightarrow \mathbf{0}$ exponentially as $t \rightarrow \infty$.

4. Dynamic-scaling-based non-certainty equivalent controller

4.1. Distributed controller design

To deal with the unknown inertia matrix

$$\mathbf{J}_i = \begin{bmatrix} J_{i11} & J_{i12} & J_{i13} \\ J_{i12} & J_{i22} & J_{i23} \\ J_{i13} & J_{i23} & J_{i33} \end{bmatrix} \quad (18)$$

a model parameter vector is defined as $\boldsymbol{\theta}_i = [J_{i11}, J_{i22}, J_{i33}, J_{i12}, J_{i13}, J_{i23}]^\top$. Then it can be easily verified that

$$\mathbf{J}_i \mathbf{x} = L(\mathbf{x}) \boldsymbol{\theta}_i \quad (19)$$

where

$$L(\mathbf{x}) = \begin{bmatrix} x_1 & 0 & 0 & x_2 & x_3 & 0 \\ 0 & x_2 & 0 & x_1 & 0 & x_3 \\ 0 & 0 & x_3 & 0 & x_1 & x_2 \end{bmatrix} \quad (20)$$

For the system defined in Eq. (16), the corresponding parameter estimation $\hat{\boldsymbol{\theta}}_i$ of $\boldsymbol{\theta}_i$ is given by

$$\hat{\boldsymbol{\theta}}_i = \boldsymbol{\beta}_i(\boldsymbol{\Phi}_i, \omega_i) + \gamma_i \quad (21)$$

where $\boldsymbol{\beta}_i$ and γ_i are two functions to be specified, and $\boldsymbol{\Phi}_i$ denotes the measurable or obtainable signals that are independent of ω_i . Therefore, the parameter estimate error becomes

$$\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\beta}_i(\boldsymbol{\Phi}_i, \omega_i) + \gamma_i - \boldsymbol{\theta}_i \quad (22)$$

Consequently, the dynamics of the parameter estimation error can be obtained by substituting Eq. (1) into the time derivative of Eq. (22) to yield the following:

$$\begin{aligned}
\dot{\tilde{\boldsymbol{\theta}}}_i &= \sum_{j=1}^n \frac{\partial \boldsymbol{\beta}_i}{\partial \Phi_{ij}} \dot{\Phi}_{ij} + \frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} \dot{\omega}_i + \dot{\gamma}_i \\
&= \sum_{j=1}^n \frac{\partial \boldsymbol{\beta}_i}{\partial \Phi_{ij}} \dot{\Phi}_{ij} + \frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} \mathbf{J}_i^{-1} (-\omega_i^\times \mathbf{J}_i \omega_i + \mathbf{u}_i) + \dot{\gamma}_i.
\end{aligned} \quad (23)$$

where denotes the j -th element of $\boldsymbol{\Phi}_i$.

Consider a regression matrix $\mathbf{W}_i \in \mathbf{R}^{3 \times 6}$, $i = 1, 2, \dots, N$

$$\begin{aligned}
\mathbf{W}_i &= -\omega_i^\times L(\omega_i) + L[(\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i)^\times \mathbf{p}_i - \mathbf{p} \mathbf{d}_i \\
&+ \frac{k_{1i}}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) (\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i) + \delta_i \bar{\omega}_i + \varphi_i \hat{\mathbf{e}}_i + k_{2i} (\bar{\omega}_i + \alpha_i \hat{\mathbf{e}}_i) \\
&+ \frac{\alpha_i}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) \bar{\omega}_i]
\end{aligned} \quad (24)$$

where δ_i , φ_i , α_i , $i = 1, 2, \dots, N$ are positive constants; $k_{2i} \in \mathbf{R}$ denotes a dynamic control gain to be determined. Hence, the time derivative of $\tilde{\boldsymbol{\theta}}_i$ can be written as

$$\begin{aligned}
\dot{\tilde{\boldsymbol{\theta}}}_i &= \dot{\gamma}_i + \sum_{j=1}^n \frac{\partial \boldsymbol{\beta}_i}{\partial \Phi_{ij}} \dot{\Phi}_{ij} + \frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} \mathbf{J}_i^{-1} (\mathbf{W}_i \boldsymbol{\theta}_i + \mathbf{u}_i) \\
&+ \frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} \left[(\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i)^\times \mathbf{p}_i - \mathbf{p} \mathbf{d}_i + \frac{k_{1i}}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) (\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i) \right. \\
&+ \delta_i \bar{\omega}_i + \varphi_i \hat{\mathbf{e}}_i + k_{2i} (\bar{\omega}_i + \alpha_i \hat{\mathbf{e}}_i) + \frac{\alpha_i}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) \bar{\omega}_i \left. \right]
\end{aligned} \quad (25)$$

Let γ_i and the control input $\mathbf{u}_i$ be assigned as

$$\begin{aligned}
\dot{\gamma}_i &= - \sum_{j=1}^n \frac{\partial \boldsymbol{\beta}_i}{\partial \Phi_{ij}} \dot{\Phi}_{ij} + \frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} [(\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i)^\times \mathbf{p}_i - \mathbf{p} \mathbf{d}_i \\
&+ \frac{k_{1i}}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) (\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i) + \delta_i \bar{\omega}_i + \varphi_i \hat{\mathbf{e}}_i + k_{2i} (\bar{\omega}_i + \alpha_i \hat{\mathbf{e}}_i) \\
&+ \frac{\alpha_i}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) \bar{\omega}_i]
\end{aligned} \quad (26)$$

$$\mathbf{u}_i = -\mathbf{W}_i (\gamma_i + \boldsymbol{\beta}_i) \quad (27)$$

Substitute Eqs. (26) and (27) into Eq. (25), and the time derivative of the parameter estimate error is simplified as

$$\dot{\tilde{\boldsymbol{\theta}}}_i = - \frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} \mathbf{J}_i^{-1} \mathbf{W}_i \tilde{\boldsymbol{\theta}}_i. \quad (28)$$

As in Ref.³⁴, an "approximate" solution $\boldsymbol{\beta}_i$ can be generated as

$$\boldsymbol{\beta}_i = \rho_i [\mathbf{W}_{i1}(\hat{\omega}_i) \omega_{ix} + \mathbf{W}_{i1}(\hat{\omega}_i) \omega_{iy} + \mathbf{W}_{i1}(\hat{\omega}_i) \omega_{iz}] \quad (29)$$

where ρ_i is a constant real number, $\mathbf{W}_i(\hat{\omega}_i)$ is the regression matrix in Eq. (24) where $\hat{\omega}_i$ is replaced by $\hat{\omega}_i$, $\mathbf{W}_{i1}(\hat{\omega}_i)$, $\mathbf{W}_{i2}(\hat{\omega}_i)$, $\mathbf{W}_{i3}(\hat{\omega}_i) \in \mathbf{R}^{6 \times 1}$ are the column vectors of $\mathbf{W}_i^T(\hat{\omega}_i)$, and the estimated angular velocity $\hat{\omega}_i$ is defined as

$$\begin{aligned}
\dot{\hat{\omega}}_i &= - \left[\delta_i \bar{\omega}_i + \varphi_i \hat{\mathbf{e}}_i + k_{2i} (\bar{\omega}_i + \alpha_i \hat{\mathbf{e}}_i) + \frac{\alpha_i}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) \bar{\omega}_i \right] \\
&- \left[(\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i)^\times \mathbf{p}_i - \mathbf{p} \mathbf{d}_i + \frac{k_{1i}}{2} (\hat{\mathbf{e}}_i^\times + \bar{\mathbf{e}}_i \mathbf{I}_3) (\bar{\omega}_i - k_{1i} \hat{\mathbf{e}}_i) \right] \\
&- \psi_i (\hat{\omega}_i - \omega_i)
\end{aligned} \quad (30)$$

where $\psi_i \in \mathbf{R}$ is a dynamic observer gain to be determined. Then partial derivative $\partial \boldsymbol{\beta}_i / \partial \omega_i$ can be obtained as

$$\frac{\partial \boldsymbol{\beta}_i}{\partial \omega_i} = \rho_i \mathbf{W}_i(\hat{\omega}_i) = \rho_i (\mathbf{W}_i + \Delta_i) \quad (31)$$

with

$$\Delta_i = \mathbf{W}_i(\hat{\omega}_i) - \mathbf{W}_i. \quad (32)$$

From the illustration in Appendix, the following inequality holds between Δ_i and the observation error $\hat{\omega}_i - \omega_i$

$$\|\Delta_i\|^2 \leq \|\bar{\Delta}_i\| \|\hat{\omega}_i - \omega_i\|^2 \quad (33)$$

The details of Δ_i and $\bar{\Delta}_i$ are given in Appendix A.2.

Substitute Eq. (31) into Eq. (28), and the time derivative of the parameter estimate error becomes

$$\dot{\hat{\theta}}_i = \rho_i (\mathbf{W}_i + \Delta_i)^T \mathbf{J}_i^{-1} (\mathbf{W}_i \hat{\theta}_i + \mathbf{u}_i) \quad (34)$$

By applying Eqs. (24) and (27), Eq. (16) can be further derived as

$$\mathbf{J}_i \dot{\hat{\omega}}_i = -\mathbf{W}_i \tilde{\theta}_i - \mathbf{J}_i \left[\delta_i \tilde{\omega}_i + \phi_i \tilde{e}_i + k_{2i} (\tilde{\omega}_i + \alpha_i \tilde{e}_i) + \frac{\alpha_i}{2} (\tilde{e}_i^\times + \bar{e}_i \mathbf{I}_3) \tilde{\omega}_i \right] + \mathbf{e}_{2i}. \quad (35)$$

Remark 5. In Eq. (34), the dynamics of $\tilde{\theta}_i$ is directly related to $\tilde{\theta}_i$, which is different to the parameter estimation scheme given in Ref.^{20,31} based on CE principle. Therefore, $\dot{\tilde{\theta}} \equiv \mathbf{0}$ holds when $\tilde{\theta} = \mathbf{0}$, i.e., once the estimation $\hat{\theta}_i$ reaches the value of θ_i , the estimation process will stop and the ideal values of inertia parameter will be held.

4.2. Dynamic scaling

Since the inertial matrix $\mathbf{J}_i$ is unknown, the minimum eigenvalue of the matrix cannot be used in controller. To overcome this difficulty, a scaled estimate error is defined as

$$\mathbf{z}_i = \frac{\exp(1/2\lambda_i^2 + 1/2)}{\exp(\sqrt{\ln r_i + 1}/\lambda_i)} \cdot \frac{\tilde{\theta}_i}{\sqrt{\lambda_i}} \quad (36)$$

where $\lambda_i > 0$ is the minimum eigenvalue of $\mathbf{J}_i$. The scaling factor is defined by the following differential equations:

$$\begin{cases} \dot{r}_i(t) = \rho_i r_i(t) \sqrt{\ln r_i(t) + 1} \|\Delta_i(t)\|^2 \\ r_i(0) = 1 \end{cases} \quad (37)$$

From Eq. (37), it can be obtained that $r_i(t) \geq 1$ holds for all $t \geq 0$. By differentiating Eq. (36) and using Eqs. (34) and (37), it follows that

$$\begin{aligned} \dot{\mathbf{z}}_i &= -\rho_i \mathbf{W}_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \rho_i \Delta_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \frac{\dot{r}_i}{2\lambda_i r_i \sqrt{\ln r_i + 1}} \mathbf{z}_i \\ &= -\rho_i \mathbf{W}_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \rho_i \Delta_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \frac{\rho_i \|\Delta_i(t)\|^2}{2\lambda_i} \mathbf{z}_i \end{aligned} \quad (38)$$

Consider a Lyapunov function

$$V_z = \frac{1}{2\rho_i} \sum_{i=1}^N \mathbf{z}_i^T \mathbf{z}_i \quad (39)$$

With Eq. (38), its derivative with respect to time is obtained as

$$\begin{aligned} \dot{V}_z &= \sum_{i=1}^N \left[-\mathbf{z}_i^T \mathbf{W}_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \mathbf{z}_i^T \Delta_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \frac{\|\Delta_i\|^2}{2\lambda_i} \mathbf{z}_i^T \mathbf{z}_i \right] \\ &\leq \sum_{i=1}^N \left[-\frac{\lambda_i}{2} \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 - \frac{\|\mathbf{z}_i\|^2}{\lambda_i} \left(\|\Delta_i\|^2 - \|\Delta_i\|^2 \right) \right] \\ &\leq -\frac{\lambda_{\min}}{2} \sum_{i=1}^N \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 \end{aligned} \quad (40)$$

where

$$\lambda_{\min} = \min\{\lambda_i, i \in (1, 2, \dots, N)\}. \quad (41)$$

Since $\mathbf{J}_i$ is positive definite, $\lambda_{\min}$ is positive too. It can be concluded that $\dot{V}_z \leq 0$, and then, $\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i$ and $\mathbf{z}_i$ are bounded.

4.3. Stability analysis

Theorem 1. Given systems (1), (2) and (5) with unknown inertia matrix $\mathbf{J}_i$, using the control law (27), the parameter estimation in Eqs. (7), (11), (12), (14), (25), (26) and (29) with the dynamic gains k_{2i} and ψ_i are defined as

$$k_{2i} = \frac{1}{2} r_i \quad (42)$$

$$\psi_i = \frac{1}{2} r_i + \frac{k_{3i}}{2} \|\bar{\Delta}_i\| + k_{4i} \quad (43)$$

where k_{3i} , k_{4i} , $i = 1, 2, \dots, N$ are positive constants. Then, the parameter estimation error $\tilde{\theta}$ is bounded, $\hat{\omega}_i(t) - \omega_i(t)$ will converge to zero, and Problem 1 is solvable.

Proof. Utilizing Eqs. (1), (24) and (30), the time derivative of $\hat{\omega}_i(t) - \omega_i(t)$ can be obtained as

$$\begin{aligned} \dot{\hat{\omega}}_i - \dot{\omega}_i &= -[\bar{\omega}_i^\times \mathbf{p}_i - \mathbf{p} \mathbf{d}_i + (\delta_i + k_{2i}) \bar{\omega}_i + (\alpha_i k_{2i} + \phi_i) \tilde{e}_i + \frac{\alpha_i}{2} (\tilde{e}_i^\times \bar{\omega}_i + \bar{e}_i \omega_e)] \\ &\quad - \psi_i (\hat{\omega}_i - \omega_i) - \mathbf{J}_i^{-1} (-\omega_i^\times \mathbf{J}_i \omega_i + \mathbf{u}_i) \\ &= \mathbf{J}_i^{-1} \mathbf{W}_i \tilde{\theta}_i - \psi_i (\hat{\omega}_i - \omega_i) \end{aligned} \quad (44)$$

Now, consider a Lyapunov-like function

$$V_1 = \frac{1}{2} \sum_{i=1}^N (\hat{\omega}_i - \omega_i)^T (\hat{\omega}_i - \omega_i) + \sum_{i=1}^N \frac{k_{3i}}{\rho_i} \sqrt{\ln r_i + 1} \quad (45)$$

Since $r_i(0) \geq 1$ due to Eq. (37), V_1 is positive definite. Using Eq. (33), Eq. (43) and the inequality

$$\begin{aligned} \exp(\sqrt{\ln r_i + 1}/\lambda_i) &\leq \exp(1/2\lambda_i^2 + \ln r_i + 1/2) \\ &\leq \exp(1/2\lambda_i^2 + 1/2) \sqrt{r_i} \end{aligned} \quad (46)$$

the time derivative of V_1 is obtained as

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^N \left(\sqrt{\lambda_i} \cdot \frac{\exp(\sqrt{\ln r_i + 1}/\lambda_i)}{\exp(1/2\lambda_i^2 + 1/2)} (\hat{\omega}_i - \omega_i)^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \psi_i \|\hat{\omega}_i - \omega_i\|^2 \right) \\ &\quad + \sum_{i=1}^N \frac{k_{3i}}{\rho_i} \cdot \frac{\dot{r}_i}{2r_i \sqrt{\ln r_i + 1}} \\ &\leq \sum_{i=1}^N \left(\frac{1}{2} \left[\frac{\exp(\sqrt{\ln r_i + 1}/\lambda_i)}{\exp(1/2\lambda_i^2 + 1/2)} \right]^2 \|\hat{\omega}_i - \omega_i\|^2 + \frac{\lambda_i}{2} \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 - \psi_i \|\hat{\omega}_i - \omega_i\|^2 \right) \\ &\quad + \sum_{i=1}^N \frac{k_{3i}}{2} \Delta_i^2 \\ &\leq \sum_{i=1}^N \left(\frac{1}{2} r_i - \psi_i + \frac{k_{3i} \|\bar{\Delta}_i\|}{2} \right) \|\hat{\omega}_i - \omega_i\|^2 + \frac{\lambda_{\max}}{2} \sum_{i=1}^N \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 \\ &\leq -\sum_{i=1}^N k_{4i} \|\hat{\omega}_i - \omega_i\|^2 + \frac{\lambda_{\max}}{2} \sum_{i=1}^N \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 \end{aligned} \quad (47)$$

where $\lambda_{\max} = \max\{\lambda_i, i \in (1, 2, \dots, N)\}$.

Next, consider another Lyapunov-like function

$$V_2 = V_1 + \frac{\lambda_{\max}}{\lambda_{\min}} V_z \quad (48)$$

Taking the time derivative of Eq. (48) yields

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \frac{\lambda_{\max}}{\lambda_{\min}} \dot{V}_z \\ &\leq -\sum_{i=1}^N k_{4i} \|\hat{\omega}_i - \omega_i\|^2 + \frac{\lambda_{\max}}{2} \sum_{i=1}^N \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 \\ &\quad - \frac{\lambda_{\max}}{\lambda_{\min}} \frac{\lambda_{\min}}{2} \sum_{i=1}^N \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 \\ &\leq -\sum_{i=1}^N k_{4i} \|\hat{\omega}_i - \omega_i\|^2 \end{aligned} \quad (49)$$

Eq. (49) indicates that $\hat{\omega}_i - \omega_i$, $\mathbf{z}_i$ and $\mathbf{r}_i$ are all bounded, and then Eq. (36) ensures that $\tilde{\theta}_i$ is also bounded.

Finally, to show the convergence of tracking errors, we consider the following Lyapunov function:

$$\begin{aligned} V_3 &= \frac{1}{2} \sum_{i=1}^N \left[(\bar{\omega}_i + \alpha_i \hat{e}_i)^T (\bar{\omega}_i + \alpha_i \hat{e}_i) + (\alpha_i \delta_i + \varphi_i) \left((\bar{e}_i - 1)^2 + \hat{e}_i^T \hat{e}_i \right) \right] \\ &\quad + \frac{\lambda_{\max}}{\lambda_{\min}} V_z \end{aligned} \quad (50)$$

Taking the derivative of V_3 in Eq. (50) and using Eqs. (35), (42) and (46), we can obtain

$$\begin{aligned} \dot{V}_3 &= \sum_{i=1}^N \left[(\bar{\omega}_i + \alpha_i \hat{e}_i)^T (-\mathbf{J}_i^{-1} \mathbf{W}_i \tilde{\theta}_i - k_{2i} (\bar{\omega}_i + \alpha_i \hat{e}_i)) - (\delta_i \|\bar{\omega}_i\|^2 + \alpha_i \varphi_i \|\hat{e}_i\|^2) + \varepsilon_{4i} \right] + \frac{\lambda_{\max}}{\lambda_{\min}} \dot{V}_z \\ &\leq \sum_{i=1}^N \left[\left(\frac{\varepsilon}{2} - k_{2i} \right) \|\bar{\omega}_i + \alpha_i \hat{e}_i\|^2 + \frac{\lambda_{\max}}{2} \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 - \delta_i \|\bar{\omega}_i\|^2 - \alpha_i \varphi_i \|\hat{e}_i\|^2 + \varepsilon_{4i} \right] - \frac{\lambda_{\max}}{2} \sum_{i=1}^N \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 \\ &\leq \sum_{i=1}^N \left[-\delta_i \|\bar{\omega}_i\|^2 - \alpha_i \varphi_i \|\hat{e}_i\|^2 + \varepsilon_{4i} \right] \end{aligned} \quad (51)$$

where

$$\begin{aligned} \varepsilon_{4i} &= (\bar{\omega}_i + \alpha_i \hat{e}_i)^T (\varepsilon_{2i} + \hat{e}_{1i}) \\ &\quad + 2[(\alpha_i \delta_i + \varphi_i) ((\bar{e}_i - 1) \bar{e}_{1i} \bar{\omega}_i + \hat{e}_i^T \hat{e}_{1i})] \end{aligned} \quad (52)$$

By Remark 4, $\hat{e}_{1i}$, ε_{2i} , $\varepsilon_{3i} \rightarrow 0$ exponentially as $t \rightarrow \infty$, then $\varepsilon_{4i} \rightarrow 0$ exponentially as $t \rightarrow \infty$. Let $\mathbf{F}(t) = \sum_{i=1}^N \varepsilon_{4i}$ and $V_4(t) = V_3(t) - \int_0^t \mathbf{F}(\tau) d\tau$, and it follows that $\mathbf{F}(t) \rightarrow 0$ as $t \rightarrow \infty$. Therefore, $\int_0^\infty \mathbf{F}(\tau) d\tau$ exists and $\int_0^t \mathbf{F}(\tau) d\tau$ is bounded for all $t \geq 0$. Then Eq. (51) ensures that $V_3(t)$ is lower bounded and leads to $\dot{V}_4 \leq \sum_{i=1}^N [-\delta_i \|\bar{\omega}_i\|^2 - \alpha_i \varphi_i \|\hat{e}_i\|^2] \leq 0$. Thus V_3 and hence V_2 are both bounded above. Therefore, it can be concluded that $\bar{\omega}_i$, $\bar{e}$ and $\bar{e}$ are all bounded.

Via Eq. (16), it is obtained that $\dot{\bar{\omega}}_i$ and $\dot{\bar{e}}_i$ are bounded, and then Eq. (50) ensures that $\dot{V}_3$ is bounded too. According to Barbalat's lemma, $\dot{V}_2$ and hence $\dot{\bar{\omega}}_i$ and $\dot{\bar{e}}_i$ will converge to zero

as $t \rightarrow \infty$. With Eq. (35), one has $\lim_{t \rightarrow \infty} \mathbf{W}_i \tilde{\theta}_i = \mathbf{0}$, $\lim_{t \rightarrow \infty} \bar{\omega}_i = \mathbf{0}$, and $\lim_{t \rightarrow \infty} \hat{e}_i = \mathbf{0}$. By Remark 3.3 in Ref.²⁰, we obtain that $\lim_{t \rightarrow \infty} \tilde{\omega}_i = \mathbf{0}$ and $\lim_{t \rightarrow \infty} \hat{e}_i = \mathbf{0}$.

Since ω_i , $\bar{\omega}_i$ and $\bar{e}$ are bounded, $\mathbf{W}_i$ and Δ_i are bounded with Eqs. (24) and (32). By Eq. (37), $\dot{r}_i$ is bounded and then by Eqs. (38) and (44), $\dot{\mathbf{z}}_i$ and $\dot{\bar{\omega}}_i - \bar{\omega}_i$ are bounded too. Thus $\dot{V}_1$ is bounded and using Barbalat's lemma again one has $\lim_{t \rightarrow \infty} \dot{V}_1 = 0$ and hence $\lim_{t \rightarrow \infty} \dot{\bar{\omega}}_i - \bar{\omega}_i = \mathbf{0}$. Therefore, $\lim_{t \rightarrow \infty} \bar{\omega}_i - \omega_i = \mathbf{0}$ according to Eq. (44). The proof is completed.

Remark 6. Although the external disturbance is not considered in Eq. (1), σ -modification can be employed to ensure the attitude tracking and parameter estimation errors against bounded external disturbance. Considering a bounded external disturbance $\mathbf{d}_i(t) \in \mathbf{R}^3$, is added to the right-hand side of Eq. (1), where $d_{\max}$ is a positive constant. Add a leakage term $-\rho_i \sigma_i(\beta_i + \gamma_i)$ to the right-hand side of Eq. (18), where σ_i is a positive constant, and then the dynamics of $\tilde{\theta}$ is given by

$$\begin{aligned} \dot{\tilde{\theta}}_i &= -\rho_i (\mathbf{W}_i + \Delta_i)^T \mathbf{J}_i^{-1} \mathbf{W}_i \tilde{\theta}_i - \rho_i \sigma_i (\tilde{\theta}_i + \theta_i) \\ &\quad + \rho_i (\mathbf{W}_i + \Delta_i)^T \mathbf{J}_i^{-1} \mathbf{d}_i \end{aligned} \quad (53)$$

Consider the same Lyapunov function V_z in Eq. (39) with the same dynamic scaling mechanism in Eq. (36), the time-derivative of V_z becomes

$$\begin{aligned} \dot{V}_z &= -\sum_{i=1}^N \left[\mathbf{z}_i^T \mathbf{W}_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \mathbf{z}_i^T \Delta_i^T \mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i - \frac{\dot{r}_i}{2\rho_i \lambda_i r_i \sqrt{\ln r_i + 1}} \mathbf{z}_i^T \mathbf{z}_i \right] \\ &\quad + \sum_{i=1}^N \left[\mathbf{z}_i^T (\mathbf{W}_i + \Delta_i)^T \mathbf{J}_i^{-1} \tilde{\mathbf{d}}_i - \sigma_i \|\mathbf{z}_i\|^2 + \sigma_i \tilde{\theta}_i^T \tilde{\theta}_i \right] \end{aligned} \quad (54)$$

where

$$\tilde{\theta}_i = \frac{\exp(1/2\lambda_i^2 + 1/2)}{\exp(\sqrt{\ln r_i + 1}/\lambda_i)} \cdot \frac{\tilde{\theta}_i}{\sqrt{\lambda_i}}, \quad \tilde{\mathbf{d}}_i = \frac{\exp(1/2\lambda_i^2 + 1/2)}{\exp(\sqrt{\ln r_i + 1}/\lambda_i)} \cdot \frac{\mathbf{d}_i}{\sqrt{\lambda_i}}$$

Let us define the scaling factor with the following derivative:

$$\dot{r}_i = 3\rho_i r_i \sqrt{\ln r_i + 1} \|\Delta_i\|^2 - k_{5i} [r_i - r_i(0)] \sqrt{\ln r_i + 1} \quad (55)$$

where k_{5i} is a positive constant. Then $\dot{V}_z$ becomes

$$\dot{V}_z \leq - \sum_{i=1}^N \left[\frac{\lambda_i}{2} \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 - \left(\frac{\sigma_i}{2} - \frac{k_{5i}}{2\lambda_i} \right) \|\mathbf{z}_i\|^2 + \left(\frac{\sigma_i \|\tilde{\boldsymbol{\theta}}_i\|^2}{2} + \frac{3\|\tilde{\mathbf{d}}_i\|^2}{2\lambda_i} \right) \right] \quad (56)$$

Eq. (55) ensures $r_i \geq r_i(0) \geq 1$. Hence, the following inequalities are established by utilizing $\exp(x) > x$:

$$\begin{aligned} \tilde{\boldsymbol{\theta}}_i &\leq \sqrt{\frac{\lambda_i \cdot \exp(1/\lambda_i^2 + 1)}{\ln r_i(0) + 1}} \cdot \boldsymbol{\theta}_i \\ \tilde{\mathbf{d}}_i &\leq \sqrt{\frac{\lambda_i \cdot \exp(1/\lambda_i^2 + 1)}{\ln r_i(0) + 1}} \cdot \mathbf{d}_i \end{aligned} \quad (57)$$

which indicate that $\tilde{\boldsymbol{\theta}}_i$ and $\tilde{\mathbf{d}}_i$ are bounded. Therefore,

$$\begin{aligned} \dot{V}_z &\leq - \sum_{i=1}^N \left[\frac{\lambda_i}{2} \|\mathbf{J}_i^{-1} \mathbf{W}_i \mathbf{z}_i\|^2 - \left(\frac{\sigma_i}{2} - \frac{k_{5i}}{2\lambda_i} \right) \|\mathbf{z}_i\|^2 \right. \\ &\quad \left. + \frac{\lambda_i \cdot \exp(1/\lambda_i^2 + 1)}{\ln r_i(0) + 1} \cdot \left(\frac{\sigma_i \|\boldsymbol{\theta}_i\|^2}{2} + \frac{3d_{\max}^2}{2\lambda_i} \right) \right] \end{aligned} \quad (58)$$

Let $\sigma_i > k_{5i}/\lambda_i$, and then the boundedness of $\mathbf{z}_i$ can be obtained according to the second term on the right-hand side of Eq. (58). Following the same design process in the proof of Theorem 1, if we chose the constant k_{3i} satisfying $k_{3i} > 1/2$, the observation error $\hat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i$ will be ultimately attracted to the set:

$$\begin{aligned} \mathcal{S}_{1i} &= \{(\hat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i) : \|\hat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i\| \\ &\leq \sqrt{\frac{d_{\max}^2}{2\lambda_i^2(k_{3i} - 1/2)} + \frac{\exp(1/\lambda_i^2 + 1)}{\ln r_i(0) + 1} \cdot \frac{\lambda_i \sigma_i \|\boldsymbol{\theta}_i\|^2 + 3d_{\max}^2}{2(k_{3i} - 1/2)}}\} \end{aligned} \quad (59)$$

Similarly, if $\phi_i > \alpha_i/2$ and $\delta_i > 1/2$, the attitude tracking errors $\hat{\mathbf{e}}_i$ and $\hat{\boldsymbol{\omega}}_i$ will also be attracted to the set:

$$\mathcal{S}_{2i} = \left\{ (\hat{\mathbf{e}}_i, \hat{\boldsymbol{\omega}}_i) : \begin{aligned} \|\hat{\mathbf{e}}_i\| &\leq \sqrt{\frac{d_{\max}^2}{\lambda_i^2 \alpha_i (\phi_i - \alpha_i/2)} + \frac{\exp(1/\lambda_i^2 + 1)}{\ln r_i(0) + 1} \cdot \frac{\lambda_i \sigma_i \|\boldsymbol{\theta}_i\|^2 + 3d_{\max}^2}{2\alpha_i (\phi_i - \alpha_i/2)}} \\ \|\hat{\boldsymbol{\omega}}_i\| &\leq \sqrt{\frac{d_{\max}^2}{\lambda_i^2 (\delta_i - 1/2)} + \exp(1/\lambda_i^2 + 1) \cdot \frac{\lambda_i \sigma_i \|\boldsymbol{\theta}_i\|^2 + 3d_{\max}^2}{2\lambda_i (\delta_i - 1/2)}} \end{aligned} \right\} \quad (60)$$

Therefore, in view of Eqs. (55), (59) and (60), $\boldsymbol{\Delta}_i$ and r_i are bounded. In addition, the size of $\mathcal{S}_{1i}$ and $\mathcal{S}_{2i}$ can be made arbitrarily small by increasing the constants k_{3i} , ϕ_i and δ_i . Moreover, if the external disturbance is slowly time-varying, it can be handled as unknown constant parameters, which will lead to the asymptotic convergence of $\hat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i$, $\hat{\mathbf{e}}_i$ and $\hat{\boldsymbol{\omega}}_i$.

5. Simulation

Consider a leader-following spacecraft system, the followers' attitude dynamics are described in Eq. (1) with the following inertia parameters:

$$\begin{aligned} \boldsymbol{\theta}_1 &= [1.2, 3.5, 4.7, 0, 0, 0]^T \\ \boldsymbol{\theta}_2 &= [1.3, 3.4, 5.2, 0, 0, 0]^T \\ \boldsymbol{\theta}_3 &= [1.9, 2.1, 3.5, 0, 0, 0]^T \\ \boldsymbol{\theta}_4 &= [2.1, 5.1, 7.1, 0, 0, 0]^T \end{aligned}$$

The leader's angular velocity $\boldsymbol{\omega}_0$ is generated by Eq. (5) with the following matrix:

$$\mathbf{S} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The simulation is conducted with the following initial conditions: $\boldsymbol{\omega}_0(0) = [2, 1, 1]^T$, $\boldsymbol{\omega}_i(0) = [0, 0, 0]^T$, $\boldsymbol{\xi}_i(0) = [0, 0, 0]^T$, $\boldsymbol{\eta}_i(0) = [0, 0, 0, 1]^T$ and $\boldsymbol{\theta}_i(0) = [0, 0, 0, 0, 0, 0]^T$ for $i = 1, 2, 3, 4$. The initial attitudes of spacecraft are given as $\mathbf{q}_0(0) = [1, 0, 0, 0]^T$, $\mathbf{q}_1(0) = [\cos(\pi/8), 0, 0, -\sin(\pi/8)]^T$, $\mathbf{q}_2(0) = [\cos(\pi/6), 0, 0, -\sin(\pi/6)]^T$, $\mathbf{q}_3(0) = [\cos(\pi/4), 0, 0, -\sin(\pi/4)]^T$ and $\mathbf{q}_4(0) = [\cos(\pi/3), 0, 0, -\sin(\pi/3)]^T$. In addition, the control gains are defined as $v_1 = 20$, $v_2 = 20$, $k_{1i} = 10$, $k_{3i} = 2$, $\alpha_i = 5$, $\delta_i = 0.5$, $\varphi_i = 10$, $\rho_i = 0.1$, $k_{4i} = 10^{-6}$ and $r_i(0) = 1$. The results based on CE in Ref.³¹ are selected as the control group, in which the control gain is given as $k_1 = 10$ and $k_2 = 10$.

The communication graph $\bar{\mathcal{G}}$ is shown in Fig. 1 and the corresponding adjacency matrix is

$$\mathbf{A}(\bar{\mathcal{G}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Simulation results of four followers are presented in Figs. 2–6. To simplify the figures, the norm of errors is used to replace the errors in these figures. Fig. 2 shows that the angular velocity error decays to zero, which indicates that the estimation angular acceleration $\hat{\mathbf{S}}_i \boldsymbol{\xi}_i(t)$ ultimately coincides with the practical angular acceleration $\mathbf{S} \boldsymbol{\omega}_0(t)$. The attitude and angular

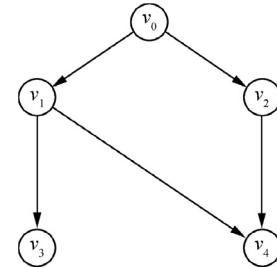


Fig. 1 Communication digraph $\bar{\mathcal{G}}$.

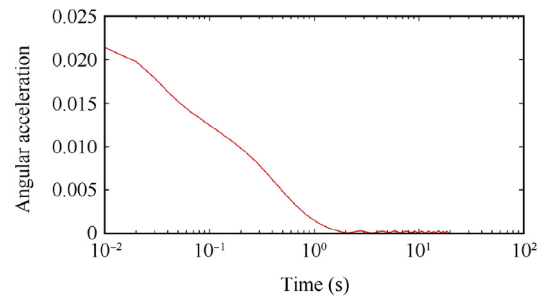


Fig. 2 Norm of angular acceleration errors $\|\hat{\mathbf{S}}_i \boldsymbol{\xi}_i(t) - \mathbf{S} \boldsymbol{\omega}_0(t)\|_2$.

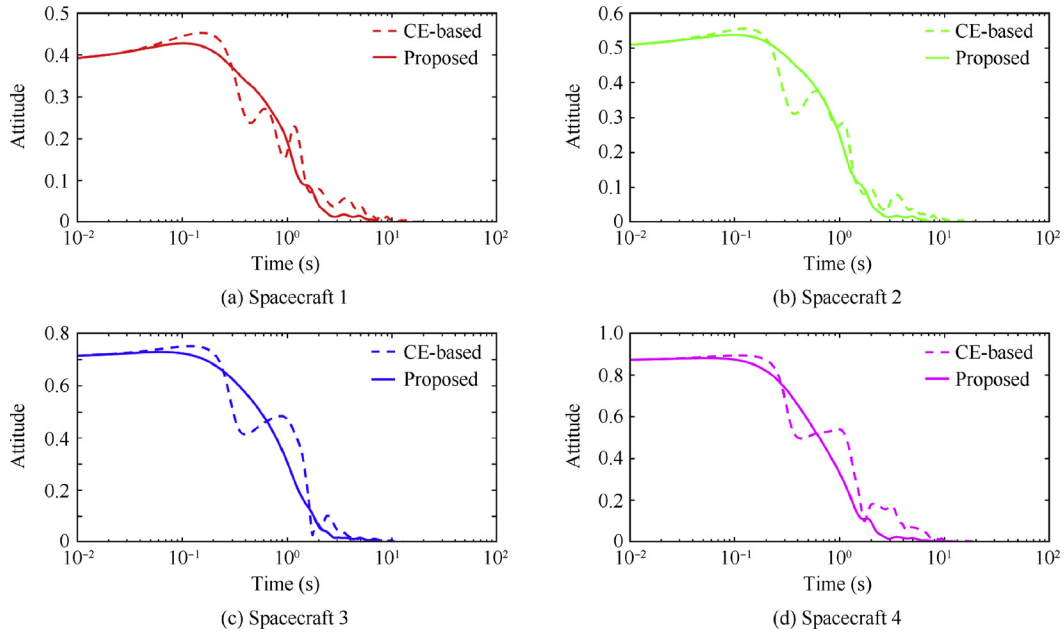


Fig. 3 Norm of attitude tracking errors $\|q_0^{-1} \odot q_i\|_2$.

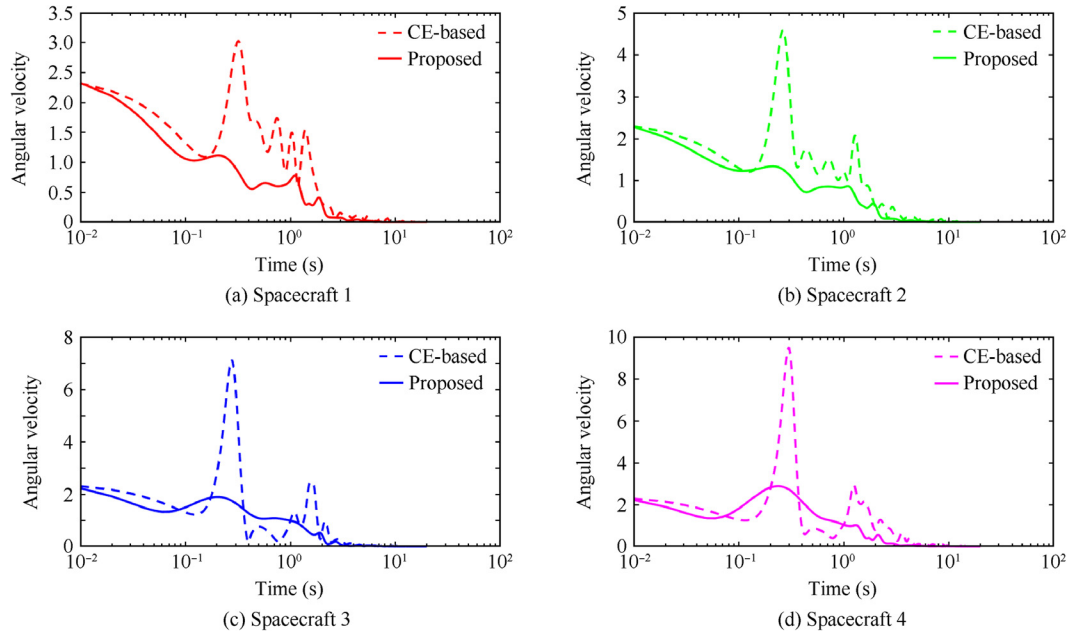


Fig. 4 Norm of angular velocity tracking errors $\|\omega_i - C(\epsilon_i)\omega_0\|_2$.

velocity tracking errors of the four followers relative to the leader are compared in Figs. 3 and 4, respectively. The results show that the errors of the attitude and angular velocity all converge to zero under both control schemes, while the convergence rate of the proposed method is obviously faster than that of the CE-based method for all the spacecraft. Fig. 5 shows the comparison of parameter estimation errors, which indicates that the proposed method converges faster than the CE-based method. The profiles of the control torque are given in Fig. 6. It can be observed in Fig. 6 that the maximum torque required by the CE-based algorithm is about three times that of the proposed controller. Since tracking of the same time-

varying reference, the control torque of steady-state for both methods are almost identical and remain time-varying.

6. Conclusion

In this paper, a dynamic-scaling-based non-certainty equivalent adaptive control scheme has been presented to address the leader-following attitude coordination problem with unknown inertia parameters. The non-certainty equivalent principle and dynamic scaling skill are used both in the distributed estimator and the distributed control law. The main advantage of the proposed method is that it does not need

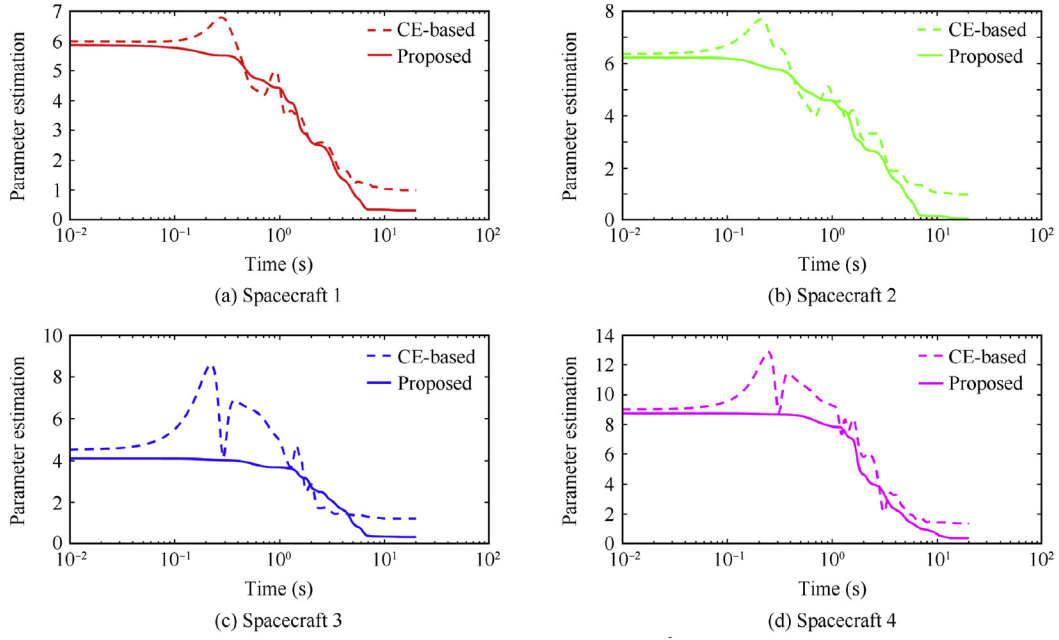


Fig. 5 Norm estimation errors $\|\hat{\theta}_i - \theta_i\|_2$.

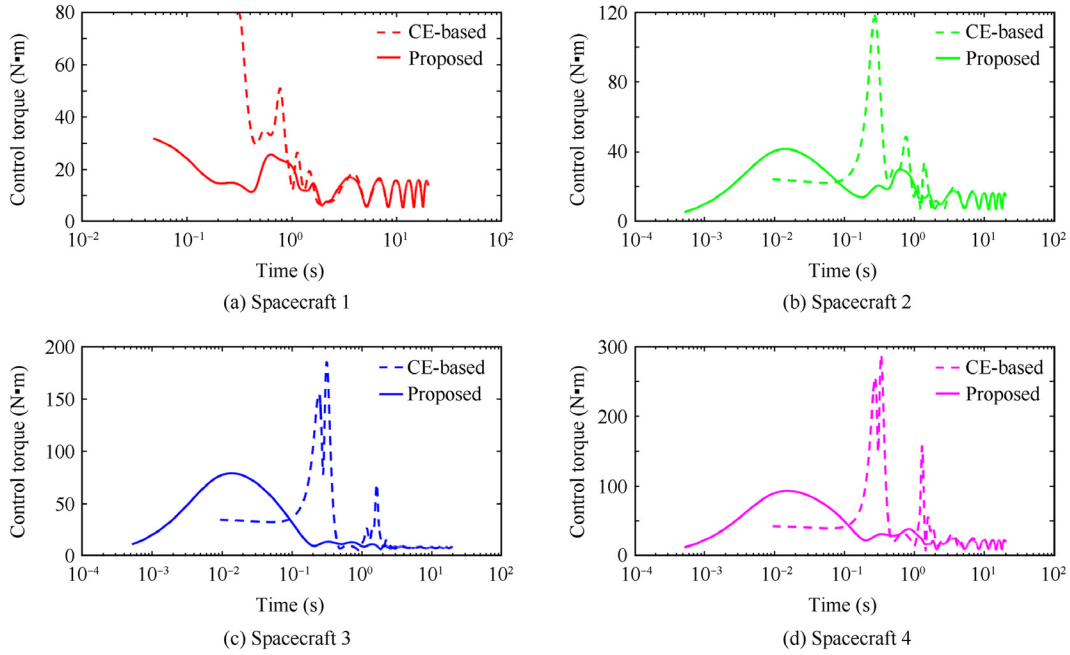


Fig. 6 Norm of control torque $\|\hat{\mu}_i\|_2$.

the boundary of inertia parameter matrix. In addition, the followers can track the attitude of the leader without any information of the leader by the proposed distributed estimation. The numerical simulations have illustrated the effectiveness of the proposed distributed control scheme. Compared with the existing schemes based on CE, our method has better convergence accuracy and faster convergence rate. Furthermore, with an upper bound of dynamic-scaling factor, the control process requires a smaller control torque to ensure the convergence of attitude and angular velocity.

Acknowledgements

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Appendix A

A.1 Proof of Lemma 1

Substituting Eq. (14) to Eq. (13), we obtain

$$\begin{aligned}
\dot{\tilde{\theta}}_S &= -\rho_S \bar{\mathbf{Q}}^T(\xi) \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S \\
&= -\rho_S \bar{\mathbf{Q}}^T(\omega_S + \xi) \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S \\
&= -\rho_S \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S - \rho_S \bar{\mathbf{Q}}^T(\xi) \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S
\end{aligned} \tag{61}$$

To eliminate the second items in Eq. (61), the dynamic scaling skill is used again. Consider a scaled parameter estimation error $\mathbf{z}_S = \tilde{\theta}_S / e^{\sqrt{\ln r_S + 1}}$ and then its derivative is given as

$$\begin{aligned}
\dot{\mathbf{z}}_S &= \frac{\dot{\tilde{\theta}}_S}{e^{\sqrt{\ln r_S + 1}}} - \frac{\dot{r}_S \tilde{\theta}}{2re^{\sqrt{\ln r_S + 1}} \sqrt{\ln r_S + 1}} \\
&= \frac{\dot{\tilde{\theta}}_S}{e^{\sqrt{\ln r_S + 1}}} - \frac{\dot{r}_S \mathbf{z}_S}{2r_S \sqrt{\ln r_S + 1}} \\
&= -\rho_S \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S - \rho_S \bar{\mathbf{Q}}^T(\xi) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S - \frac{\dot{r}_S \mathbf{z}_S}{2r_S \sqrt{\ln r_S + 1}}
\end{aligned} \tag{62}$$

Define a Lyapunov function

$$V_{z_S} = \frac{1}{2\rho_S} \mathbf{z}_S^T \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \tag{63}$$

and then its time derivative is obtained as

$$\begin{aligned}
\dot{V}_{z_S} &= -\mathbf{z}_S^T \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \\
&\quad - \mathbf{z}_S^T \bar{\mathbf{Q}}^T(\xi) \bar{\mathbf{Q}}(\omega_S) \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \\
&\quad - \frac{\dot{r}_S}{2\rho_S r_S \sqrt{\ln r_S + 1}} \mathbf{z}_S^T \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \\
&\leq -\frac{1}{2} \mathbf{z}_S^T \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \\
&\quad + \left(\frac{\|\bar{\mathbf{Q}}(\xi)\|^2}{2} - \frac{\dot{r}_S}{2\rho_S r_S \sqrt{\ln r_S + 1}} \right) \|\bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S\|^2
\end{aligned} \tag{64}$$

where the Young inequality has been employed. Let

$$\dot{r}_S = \rho_S r_S \sqrt{\ln r_S + 1} \|\bar{\mathbf{Q}}(\xi)\|^2 \tag{65}$$

Replacing Eq. (65) in Eq. (64) yields

$$\dot{V}_{z_S} \leq -\frac{1}{2} \mathbf{z}_S^T \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \tag{66}$$

Since ω_0 is a persistent excitation signal, the matrix $\bar{\mathbf{Q}}(\omega_S)$ is row full rank. Therefore, there exist some positive constants α_S such that $\bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) > \alpha_S \mathbf{I}_{3N}$, and then Eq. (66) can be rewritten as

$$\dot{V}_{z_S} \leq -\frac{\alpha_S}{2} \mathbf{z}_S^T \bar{\mathbf{Q}}^T(\omega_S) \bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S = -\alpha_S V_{z_S} \tag{67}$$

Therefore, $\bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S \rightarrow 0$ exponentially as $t \rightarrow \infty$. Similar as the construction of Lyapunov-like function Eq. (45) of Theorem 1 and the following deduction, we have r_S , $\mathbf{z}_S$ and ξ which are all bounded. Therefore,

$$\|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\| = \|\bar{\mathbf{Q}}(\omega_S) r_S \mathbf{z}_S\| \leq r_{\max} \|\bar{\mathbf{Q}}(\omega_S) \mathbf{z}_S\| \rightarrow 0 \text{ as } t \rightarrow \infty. \tag{68}$$

Theorem 2.5.3 in Ref.³⁵ guarantees the existence of a positive definite diagonal matrix $\mathbf{C} = \text{diag}\{c_1, c_2, \dots, c_N\}$ such that $\bar{\mathbf{H}} = \mathbf{C}\mathbf{H} + \mathbf{H}^T \mathbf{C}$ is positive definite. Let $\bar{c} = \max\{c_1, c_2, \dots, c_N\}$ and be the smallest eigenvalue of $\bar{\mathbf{H}}$. Let $V_{1S} = \frac{1}{2} \mathbf{y}^T \mathbf{y} + \frac{1}{2\alpha_S} \bar{\theta}_S^T \bar{\mathbf{Q}}(\omega_S) \bar{\mathbf{Q}}^T(\omega_S) \tilde{\theta}_S$

$$U = \frac{1}{2} \varpi \boldsymbol{\zeta}^T \mathbf{P} \boldsymbol{\zeta} + \boldsymbol{\zeta}^T (\mathbf{C} \otimes \mathbf{I}_4) \boldsymbol{\zeta} + \frac{1}{2\alpha_S} \bar{\theta}_S^T \bar{\mathbf{Q}}(\omega_S) \bar{\mathbf{Q}}^T(\omega_S) \tilde{\theta}_S \tag{69}$$

where $\mathbf{P}$ is a positive definite matrix such that $\mathbf{P}\mathbf{E} + \mathbf{E}^T \mathbf{P} = -\mathbf{I}_{3N}$ and $\bar{c} > 2 + \bar{c}^2 N^2 / v_1 \gamma$.

By direct calculating, we have

$$\dot{U} = -\varpi \|\boldsymbol{\zeta}\|^2 - \boldsymbol{\zeta}^T (v_1 \bar{\mathbf{H}} \otimes \mathbf{I}_4) \boldsymbol{\zeta} - \frac{\varpi}{\alpha_S} \|\bar{\mathbf{Q}}(\omega_S)^T \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \omega(t) \tag{70}$$

where

$$\begin{aligned}
\omega(t) &= \varpi \boldsymbol{\zeta}^T \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S + \sum_{i=1}^N c_i \xi_i q_0 \odot q(\xi_i) \\
&\leq \frac{\varpi}{2} \|\boldsymbol{\zeta}\|^2 + \frac{\varpi}{2} \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \sum_{i=1}^N \left(\frac{v_1}{2N} \|\xi_i\|^2 + \frac{c_i^2 N}{2v_1} \|q_0 \odot q(\xi_i)\|^2 \right) \\
&= \frac{\varpi}{2} \|\boldsymbol{\zeta}\|^2 + \frac{\varpi}{2} \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \sum_{i=1}^N \left(\frac{v_1}{2N} \|\xi_i\|^2 + \frac{c_i^2 N}{2v_1} \|q(\xi_i)\|^2 \right) \\
&= \frac{\varpi}{2} \|\boldsymbol{\zeta}\|^2 + \frac{1}{2} \varpi \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \sum_{i=1}^N \left(\frac{v_1}{2N} \|\xi_i\|^2 + \frac{c_i^2 N}{2v_1} \|\xi_i\|^2 \right) \\
&\leq \frac{\varpi}{2} \|\boldsymbol{\zeta}\|^2 + \frac{\varpi}{2} \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \sum_{i=1}^N \left(\frac{v_1}{2N} \|\xi_i\|^2 + \frac{c_i^2 N}{2v_1} \|\xi_i\|^2 \right) \\
&= \frac{\varpi}{2} \|\boldsymbol{\zeta}\|^2 + \frac{\varpi}{2} \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \frac{v_1}{2} \|\boldsymbol{\zeta}\|^2 + \frac{c^2 N^2}{2v_1} \|\boldsymbol{\zeta}\|^2
\end{aligned} \tag{71}$$

In the derivation of Eq. (71), the identity $\|\mathbf{q}_0 \odot \mathbf{q}(\xi_i)\| = \|\mathbf{q}(\xi_i)\|$ implied by Lemma 2 is employed. Substituting (71) into (70) gives

$$\begin{aligned}
\dot{U} &\leq -\varpi \|\boldsymbol{\zeta}\|^2 - v_1 \|\boldsymbol{\zeta}\|^2 - \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \omega(t) \\
&= -\varpi \|\boldsymbol{\zeta}\|^2 - v_1 \|\boldsymbol{\zeta}\|^2 - \varpi \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \frac{\varpi}{2} \|\boldsymbol{\zeta}\|^2 \\
&\quad + \frac{\varpi}{2} \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 + \frac{v_1}{2} \|\boldsymbol{\zeta}\|^2 + \frac{c^2 N^2}{2v_1} \|\boldsymbol{\zeta}\|^2 \\
&\leq -\|\boldsymbol{\zeta}\|^2 - \frac{v_1}{2} \|\boldsymbol{\zeta}\|^2 - \frac{\varpi}{2} \|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|^2 \leq -\alpha_U U
\end{aligned} \tag{72}$$

for some $\alpha_U > 0$. Hence, it can be obtained that $\|\boldsymbol{\zeta}\|$, $\|\xi\|$ and $\|\bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S\|$ converge to zero exponentially. Therefore, $\lim_{t \rightarrow \infty} (\tilde{\mathbf{S}}_i - \mathbf{S}) \xi_i(t) = \lim_{t \rightarrow \infty} \tilde{\mathbf{S}}_i(\omega_0(t) + \xi_i(t)) = \lim_{t \rightarrow \infty} \bar{\mathbf{Q}}(\omega_S) \tilde{\theta}_S + \tilde{\mathbf{S}}_i \xi_i(t) = \mathbf{0}$, $\lim_{t \rightarrow \infty} (\boldsymbol{\eta}_i(t) - \mathbf{q}_0(t)) = \mathbf{0}$ and $\lim_{t \rightarrow \infty} (\hat{\xi}_i(t) - \omega_0(t)) = \mathbf{0}$ exponentially.

A.2 Details of Δ_i and $\bar{\Delta}_i$

In this part, the detailed form of Δ_i and $\bar{\Delta}_i$ will be given to complete the controller derivation. For notational convenience, we

define $\tilde{\boldsymbol{\theta}}_i = \hat{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i = [\tilde{\boldsymbol{\omega}}_{ix}, \tilde{\boldsymbol{\omega}}_{iy}, \tilde{\boldsymbol{\theta}}_{iz}]^T$, $\kappa_i = (k_{1i} + \alpha_i)/2$, $\hat{\mathbf{e}}_i - \mathbf{p}_i = [\kappa_{ix}, \kappa_{iy}, \kappa_{iz}]^T$ and $\mu_i = (k_{1i} + \alpha_i)/2 \cdot \bar{e}_i + \delta_i + k_{2i}$. By applying Eqs. (14), (16) and (18), Δ_i can be obtained as

$$\begin{aligned}
\Delta_i &= W(\hat{\boldsymbol{\omega}}_i) - W(\boldsymbol{\omega}_i) \\
&= -\hat{\boldsymbol{\omega}}_i L(\tilde{\boldsymbol{\omega}}_i) - \tilde{\boldsymbol{\omega}}_i^* L(\boldsymbol{\omega}_i) \\
&\quad + L \left[\tilde{\boldsymbol{\omega}}_i^* \mathbf{p}_i + \frac{k_{1i} + \alpha_i}{2} (\hat{\mathbf{e}}_i^* + \bar{\mathbf{e}} \mathbf{I}) \tilde{\boldsymbol{\omega}}_i + (\delta_i + k_{2i}) \tilde{\boldsymbol{\omega}}_i \right] \\
&= -\hat{\boldsymbol{\omega}}_i L(\tilde{\boldsymbol{\omega}}_i) - \tilde{\boldsymbol{\omega}}_i^* L(\boldsymbol{\omega}_i) + L(\kappa_i^* \tilde{\boldsymbol{\omega}}_i) + \mu_i L(\tilde{\boldsymbol{\omega}}_i)
\end{aligned} \tag{73}$$

The details are given below:

$$\hat{\omega}_i^\times L(\hat{\omega}_i) = \begin{bmatrix} 0 & \hat{\omega}_{iz}\hat{\omega}_{ix} & -\hat{\omega}_{iy}\hat{\omega}_{ix} \\ -\hat{\omega}_{iz}\hat{\omega}_{iy} & 0 & \hat{\omega}_{ix}\hat{\omega}_{iy} \\ \hat{\omega}_{iy}\hat{\omega}_{iz} & -\hat{\omega}_{ix}\hat{\omega}_{iz} & 0 \\ -\hat{\omega}_{iz}\hat{\omega}_{ix} & \hat{\omega}_{iz}\hat{\omega}_{iy} & \hat{\omega}_{ix} - \hat{\omega}_{iy}\hat{\omega}_{iy} \\ \hat{\omega}_{iy}\hat{\omega}_{ix} & \hat{\omega}_{iz}\hat{\omega}_{iz} - \hat{\omega}_{ix}\hat{\omega}_{ix} & -\hat{\omega}_{iy}\hat{\omega}_{iz} \\ \hat{\omega}_{iy}\hat{\omega}_{iy} - \hat{\omega}_{iz}\hat{\omega}_{iz} & -\hat{\omega}_{ix}\hat{\omega}_{iy} & \hat{\omega}_{ix}\hat{\omega}_{iz} \end{bmatrix}^T \quad (74)$$

$$\tilde{\omega}_i^\times L(\tilde{\omega}_i) = \begin{bmatrix} 0 & \tilde{\omega}_{iz}\tilde{\omega}_{ix} & -\tilde{\omega}_{iy}\tilde{\omega}_{ix} \\ -\tilde{\omega}_{iz}\tilde{\omega}_{iy} & 0 & \tilde{\omega}_{ix}\tilde{\omega}_{iy} \\ \tilde{\omega}_{iy}\tilde{\omega}_{iz} & -\tilde{\omega}_{ix}\tilde{\omega}_{iz} & 0 \\ -\tilde{\omega}_{iz}\tilde{\omega}_{ix} & \tilde{\omega}_{iz}\tilde{\omega}_{iy} & \tilde{\omega}_{ix}\tilde{\omega}_{ix} - \tilde{\omega}_{iy}\tilde{\omega}_{iy} \\ \tilde{\omega}_{iy}\tilde{\omega}_{ix} & \tilde{\omega}_{iz}\tilde{\omega}_{iz} - \tilde{\omega}_{ix}\tilde{\omega}_{ix} & -\tilde{\omega}_{iy}\tilde{\omega}_{iz} \\ \tilde{\omega}_{iy}\tilde{\omega}_{iy} - \tilde{\omega}_{iz}\tilde{\omega}_{iz} & -\tilde{\omega}_{ix}\tilde{\omega}_{iy} & \tilde{\omega}_{ix}\tilde{\omega}_{iz} \end{bmatrix}^T \quad (75)$$

$$L(\kappa_i^\times \tilde{\omega}_i) = \begin{bmatrix} \kappa_{iy}\tilde{\omega}_{iz} - \kappa_{iz}\tilde{\omega}_{iy} & 0 & 0 \\ 0 & \kappa_{iz}\tilde{\omega}_{ix} - \kappa_{ix}\tilde{\omega}_{iz} & 0 \\ 0 & 0 & \kappa_{ix}\tilde{\omega}_{iy} - \kappa_{iy}\tilde{\omega}_{ix} \\ \kappa_{iz}\tilde{\omega}_{ix} - \kappa_{ix}\tilde{\omega}_{iz} & \kappa_{iy}\tilde{\omega}_{iz} - \kappa_{iz}\tilde{\omega}_{iy} & 0 \\ \kappa_{ix}\tilde{\omega}_{iy} - \kappa_{iy}\tilde{\omega}_{ix} & 0 & \kappa_{iy}\tilde{\omega}_{iz} - \kappa_{iz}\tilde{\omega}_{iy} \\ 0 & \kappa_{ix}\tilde{\omega}_{iy} - \kappa_{iy}\tilde{\omega}_{ix} & \kappa_{iz}\tilde{\omega}_{ix} - \kappa_{ix}\tilde{\omega}_{iz} \end{bmatrix}^T \quad (76)$$

$$L(\tilde{\omega}_i) = \begin{bmatrix} \tilde{\omega}_{ix} & 0 & 0 \\ 0 & \tilde{\omega}_{iy} & 0 \\ 0 & 0 & \tilde{\omega}_{iz} \\ \tilde{\omega}_{iy} & \tilde{\omega}_{ix} & 0 \\ \tilde{\omega}_{iz} & 0 & \tilde{\omega}_{ix} \\ 0 & \tilde{\omega}_{iz} & \tilde{\omega}_{iy} \end{bmatrix}^T \quad (77)$$

Since all elements of Δ_i in Eqs. (73)(77) can be written as $\tilde{\omega}^T \theta$, where $\Theta \in \mathbf{R}^3$, Δ_i can be given in a compact form as

$$\Delta_i = \begin{bmatrix} \tilde{\omega}_i & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \tilde{\omega}_i & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \tilde{\omega}_i \end{bmatrix}^T K \quad (78)$$

where denotes the corresponding matrix. Then one has

$$\Delta_i \Delta_i^T = \begin{bmatrix} \tilde{\omega}_i & \mathbf{0}_{3 \times 1} & \mathbf{0} \\ \mathbf{0}_{3 \times 1} & \tilde{\omega}_i & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \tilde{\omega}_i \end{bmatrix}^T \bar{\Delta}_i \begin{bmatrix} \tilde{\omega}_i & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \tilde{\omega}_i & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \tilde{\omega}_i \end{bmatrix} \quad (79)$$

with

$$\bar{\Delta}_i = K K^T \quad (80)$$

It can be concluded for the definition of 2-norm that

$$\|\Delta_i \Delta_i^T\| = \|\Delta_i\|^2 \leq \|\bar{\Delta}_i\| \|\hat{\omega} - \omega\|^2 \quad (81)$$

Thereby the detailed forms of Δ_i and $\bar{\Delta}_i$ are given.

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